K & μ Values – Elongation Calculation

Question:

How wobble & friction affect the elongation of a tendons?

Answer:

The calculation of losses due to friction is a major issue both in design and construction of post tensioned structures. European and American standards implement a different approach to estimate those losses.

AASHTO LRFD (Bridge Design Specifications eq. 5.9.5.2.2b-1):

Losses due to friction for internal tendons may be taken as:

$$\Delta f_{Pf} = f_{pj}(1-e^{-(Kx+\mu\alpha)})$$

where:

f_{pj}	stress in the prestressing steel at jacking (MPa)
x	length of a prestressing tendon from the jacking end to any point under consideration (mm)
K	wobble friction coefficient (per mm of tendon)
α	sum of the absolute values of angular change of prestressing steel path from jacking end, or from the nearest jacking end if tensioning is done equally at both ends, to the point under investigation (rad)
μ	coefficient of friction

Eurocodes (EN 1992-1-1, eq. 5.45):

Losses due to friction in post-tensioned tendons may be estimated from:

$$\Delta P_{\mu}(x) = P_{max}(1-e^{-\mu(\theta+kx)})$$

where:

P	Prestressing force
θ	the sum of the angular displacements over distance x
μ	coefficient of friction between the tendon and its duct
k	unintentional angular displacement for internal tendons
x	distance along the tendon from the point where the prestressing force is equal to Pmax

In conclusion, If the equation in form $e^{-(\kappa x+\mu\alpha)}$ is used, instead of $e^{-\mu(\kappa x+\alpha)}$, we consider that, the relationship between the factors contained denotes that, $K = \mu\kappa$.

The Theory:

1. Both standards, AASHTO & EN are considering that the values of μ & κ/K are part of the PT system implemented in the structure, in other words you have to input the values given by each PT system.

2. However, each PT system is using/recommending different values of μ & κ/K.

3. During initial design stage, the Designer/structural Engineer does not know which PT system will be used.

4. The elongation studies, either are using the values of a specific PT system or may consider the recommended values specified in both standards in case of absence of any data.

5. Recommended values according to AASHTO LRFD

TYPE of STRAND & DUCT	μ	K (AASHTO)
	rad^{-1}	ft^{-1}
Internal tendons-Steel corrugated ducts	0.15-0.25	0.0002
Internal tendons-Plastic ducts	0.23	0.0002
External tendons-Steel deviators	0.25	0.0002

6. Recommended values according to PTI/ASBI M50.3

TYPE of PRESTRESSING STEEL	CORRUGATED METAL DUCT	CORRUGATED PLASTIC DUCT	SMOOTH STEEL PIPE	SMOOTH PLASTIC PIPE
	μ/k (ft^{-1})	μ/k (ft^{-1})	μ/k (ft^{-1})	μ/k (ft^{-1})
Strand	0.15 to 0.25/ 0.00005 to 0.0003	0.10 to 0.14/ 0.00005 to 0.0003	0.25 to 0.30/ 0	0.10 to 0.14/ 0
Strand in precast elements and constant curvature tendons	0.15 to 0.25/ 0.00005 to 0.0003	0.10 to 0.14/ 0.00005 to 0.0003	--	--
External tendons, bare dry strand	--	--	0.25 to 0.30/ 0	0.12 to 0.15/ 0
Lubricated strand	0.12 to 0.18/ 0.00005 to 0.0003	--	0.20 to 0.25/ 0	--
Strand greased and extruded	0.01 to 0.05/ 0.00005 to 0.0003	0.01 to 0.05/ 0.00005 to 0.0003	0.01 to 0.05/ 0	0.01 to 0.05/ 0
Bars, deformed, smooth and round	0.30/ 0 to 0.0002	0.30/ 0 to 0.0002	--	--

7. Recommended values according to EN-1992-1-1

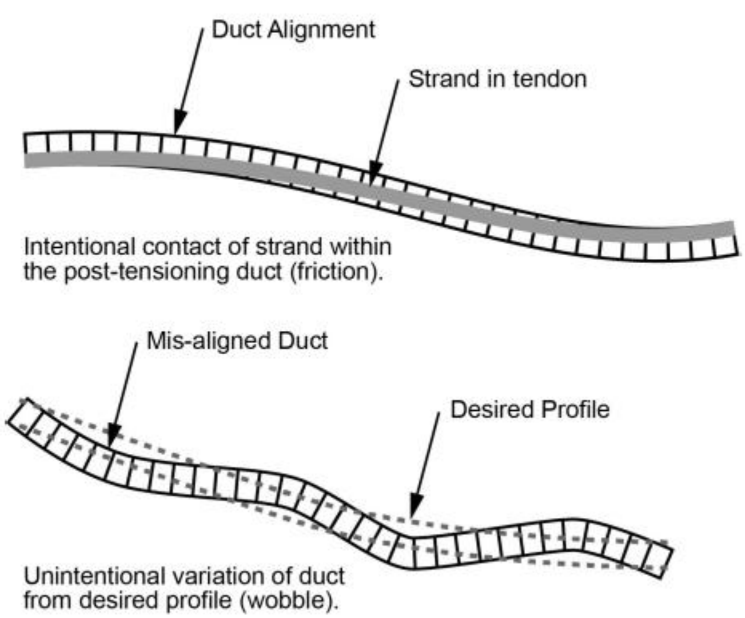
TYPE of STRAND & DUCT	μ non-lubricated	μ lubricated	κ (EN)
	rad^{-1}	rad^{-1}	rad/m
Internal tendons-Steel corrugated ducts	0.19	--	0.005-0.01
External tendons-Steel deviators	0.24	0.16	
External tendons-Plastic deviators	0.12	0.10	

8. All these values mentioned in the standards or given by the PT system are RECOMMENDED, meaning that the Designer/structural Engineer is free to use any value according to his engineering judgement.

The Facts:

a. The friction coefficient μ depends on various factors such as inaccurate placement of PT tendon, improper placement of strands or damaged sheathing, bending radius, strands and sheaths nature, quality of contact surfaces etc.

b. The wobble coefficient (K/κ) is related with sloppy placement or excessive tendon deviations, stiffness of ducts, distances between tendons supports, vibrations during concreting etc.



c. The supply of strands & ducts in most of the cases is out of the scope and control of the PT system (different factories, different manufacturers-suppliers, different material properties etc). The PT system cannot force any Owner, Client, Contractor etc. to use specific strands and ducts. According to our knowledge there is none standard specifying such things since there are so many approved and certified manufacturers – suppliers of strands and ducts worldwide.

d. There is a discrepancy between the elongation study values of the initial design (based on the engineering judgement of the designer) and those of the PT system which will be implemented in a structure (based on the recommended values of each specific PT system). The figures cannot be match considering also different structural software/modelling used.

e. It is also a fact that no matter what value of μ and K/κ will be adopted in the calculations, the differences seems to be negligible, within a reasonable range. However, in cases of long tendons the differences in elongations may become significant.

Conclusions:

Question:

We have been asked many times to justify why the elongation studies of the PT system are different from those specified by the initial design.

Answer:

For all above reasons. Different equations (AASHTO-EN), different values of μ & K/κ, different software/modelling assumptions are impossible to give same results.

Question:

Should Consultants/Engineers be aware of such differences?

Answer:

No. The PT installer has the responsibility to study and provide the theoretical elongations based on the recommended values of the used PT system.

Question:

During the stressing stage, the field elongations should be checked and compared with the theoretical elongations. What is the theoretical elongation we have to consider for the comparison?

Answer:

Usually, there are three (3) theoretical elongation studies:

a. From the initial design (considering a theoretical value of modulus of elasticity of the strand, usually E=195 GPa and engineering judgement values of μ, κ/K)
b. From the elongation studies of the PT installer (considering E=195 GPa and the recommended values of the used PT system μ, κ/K)
c. From the elongation studies of the PT installer (considering the actual E mentioned in the strand coil tag and the recommended values of the used PT system μ, κ/K)

Based on the field experience all these elongations studies are giving different figures but close to each other. The E values have a much stronger impact than μ, κ/K values.

The field elongations during stressing should be compared with the elongation studies of the PT installer (item b above) and usually should be within a range of:

EN-13670-Execution of Concrete Structures/par. 7.6.1
±5% of the specified total tensioning force or within ±10% of the specified tensioning force for a single tendon
AASHTO LRFD Bridge Construction Specifications/par. 10.10.1.4
±7% of the gage pressure and elongation in tendons exceeding 50ft (15m) long

Item c elongation studies are useful to know a more accurate value about the expected field elongation.

Question:

Is the elongation so important?

Answer:

What is important is the stressing force specified by the design. The elongation is a way to check that the proper stressing force has been applied in the tendon (a back-up system to confirm the stressing force). There are so many above mentioned parameters which affect the elongation and expected to be within the range of ±5% or (PTI specifies 7% even 10% especially for short tendons) of the design values.