ELONGATION - MODULUS OF ELASTICITY CORRELATION

According to Hooke's Law the modulus of Elasticity E and the strain ε are inversely proportional considering invariable the stress σ of a linear elastic system:

$$\sigma = E \cdot \varepsilon \,, \quad (1.1)$$

For a post-tensioned tendon, the elongation ΔL is calculated as follows:

$$\varepsilon = \frac{\Delta L}{L} \,, \quad (1.2)$$

$$\varepsilon = \frac{\sigma}{E} \therefore \frac{\Delta L}{L} = \frac{\sigma}{E} \therefore \Delta L = \frac{\sigma}{E} L \,, \quad (1.3)$$

The stress σ is not stable through the whole length of the tendon because of the friction losses that are result of the curvature effect and the wobble effect. So, an integration of the stresses is necessary for the estimation of the elongation according to the following formula:

$$\Delta L = \int_0^L \frac{\sigma}{E} dx = \frac{1}{E} \int_0^L \sigma dx \,, \quad (1.4)$$

The stress at a random point x of the tendon is:

$$\sigma_x = \sigma_i \cdot e^{-(\mu\alpha + Kx)} \,, \quad (1.5)$$

where :

σ_i is the stress at a previous point or the anchorage

x is the distance from that previous point or the anchorage

a is the angle of deviation (rad) between the previous point or the anchorage

μ is the friction coefficient between the strand and the sheath (1/rad)

K is the coefficient of the unintentional angular deviation (rad).

In case there is no change in the geometry of the tendon, the PT system that is used and the tensioning force, the stresses do not change for each distance x. Therefore, if the only parameter that changes is the Modulus of Elasticity, according to the equation (1.4) the integration term remains stable, and so the elongation is inversely proportional to the Modulus of Elasticity :

$$\Delta L \cdot E = c, c = stable \,, \quad (1.6)$$

As an example, we consider a 4-strand flat tendon with length L, initial design modulus of Elasticity $E_1 = 195MPa$, tensioning force F and the calculated elongation $\Delta L = 77.5mm$. After the dispatch of the coil strand on site, the modulus of Elasticity provided by the manufacturer is equal to $E_2 = 196.2MPa$. According to the equation (1.6) the new and more accurate elongation can be easily calculated as follows:

$$\Delta L_1 \cdot E_1 = \Delta L_2 \cdot E_2 \therefore \Delta L_2 = \Delta L_1 \frac{E_1}{E_2} \quad (1.7)$$

, thus ΔL2 = 77.5 x (195/196.2) = 77.03mm

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